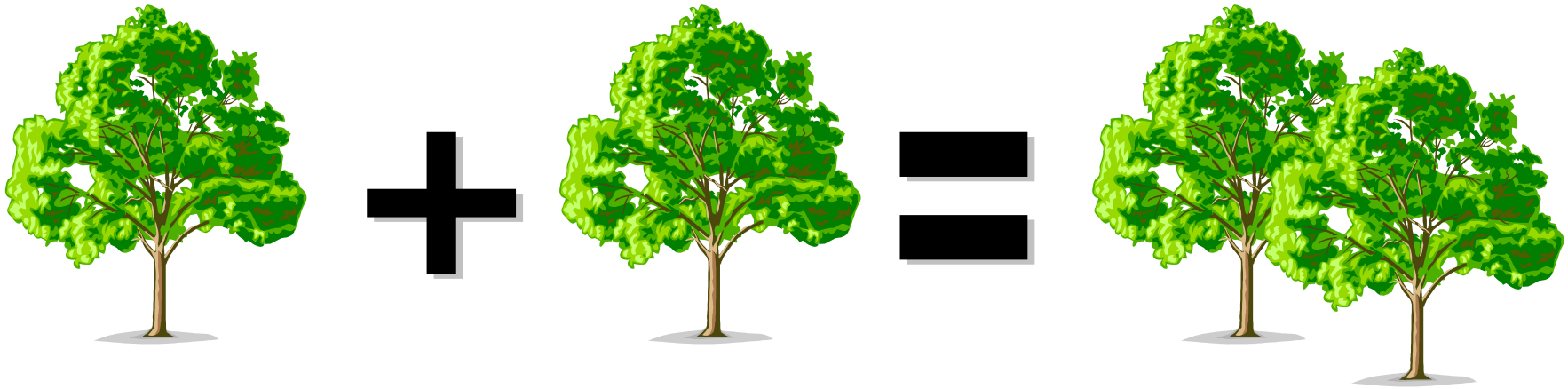
Nook Lane Junior School



Calculations' Policy

Progression towards a standard written method of Calculation

Introduction

Children are introduced to the process of Calculation through practical, oral and mental activities. Through these activities they consolidate their understanding of number facts and begin to develop ways of recording to support their thinking and calculation methods.

As children's informal methods are strengthened they become ready to use more efficient written methods.

It is our aim that, by the end of Year 6, children will be equipped with mental, written and calculator methods that they understand and use correctly and, when faced with a calculation, they should be able to decide which method is the most appropriate and have strategies to check its accuracy.

This guidance focuses on the use of 'standard' written methods – compact methods that are efficient and work for any calculations, including those that involve whole numbers and decimals.

Being able to use these written methods gives children an efficient set of tools that they can use when they are unable to carry out the calculation in their heads or with a calculator.

The guidance covers written methods for each of the number operations (addition, subtraction, multiplication and division) showing the progression in stages and compliments the Parents' Booklet.

Why do we need this policy?

- Consistency in methods taught throughout the school
- Progression from informal / practical methods of recording to written methods for each of the four operations.
- An aid to parents' understanding in their child's stages of learning.

Things to remember

- Children should always estimate first.
- Give time to check the answer, preferably using a different method eg. the inverse operation.
- Always decide first whether a mental method is appropriate.
- Pay attention to mathematical language – use the correct terms.
- Children who make persistent mistakes should return to the method that they can use accurately until ready to move on.
- Children need to know number and multiplication facts by heart.
- When revising or extending to harder numbers - refer back to expanded methods. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.
- Use practical equipment if necessary when introducing or reviewing a concept.
- Children should be introduced to calculators from Y3, developing skills to use them effectively in Years 4 to 6.

Equality and Diversity

Nook Lane Junior School is committed to providing an environment which embraces diversity and promotes equality of opportunity (see Diversity & Equality policy statement) and we believe that all children should have equal access and inclusive rights to the Mathematics regardless of their gender, race, disability or ability.

We plan work that is differentiated for the performance of all groups and individuals, ensuring that activities are targeted to match ability and work closely with anyone supporting a child with special needs.

We are committed to creating a positive climate that will enable everyone to work free from racial intimidation and harassment and to achieve their full potential.

Objectives

At Nook Lane we follow the primary Framework for Mathematics, published by the Department for Education and Skills.

The objectives in the framework show the progression in children's use of written methods of calculation in the strands 'Using and Applying Mathematics' and 'Calculating'.

Using and Applying	Calculating
Year 3 • Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations • Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	**Year 3** • Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers • Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences
Year 4 • Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate • Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	**Year 4** • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6)
Year 5 • Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use • Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	**Year 5** • Use efficient written methods to add and subtract whole numbers and decimals with up to two places • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 • Refine and use efficient written methods to multiply and divide HTU × U, TU × TU, U.t × U and HTU ÷ U
Year 6 • Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use • Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of *c* pens at 15 pence each is 15*c* pence)	**Year 6** • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

Progression in written methods for addition

These notes show the stages in building up to a compact, efficient method for addition. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to:

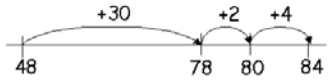
- recall addition pairs to 9 + 9
- know all complements to 10
- add mentally a series of single-digit numbers, such as 5 + 8 + 4
- count on in 1s, 10s and 100s
- partition numbers in ways other than into tens and ones to help with bridging multiples of 10 and 100

Empty number line

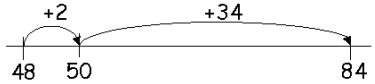
The empty number line helps to record the steps on the way to calculating the total.
The steps often bridge through a multiple of 10.

Example:

48 + 36 = 84



or:



Children need to be able to:

- partition numbers into hundreds, tens and ones
- recall addition pairs to 9 + 9
- add multiples of 10 or 100 (such as 60 + 70 or 600 + 700) using a related fact (6 + 7) and knowledge of place value
- mentally add multiples of 100, 10 and 1 e.g. 800 + 130 + 12

Partitioning

When adding larger numbers, it becomes less efficient to count on so partitioning is used. Partition into (hundreds) tens and ones, add to form partial sums and then recombine.

Partitioning all the numbers mirrors the standard column method where ones are placed under ones and tens under tens etc.

Example:

Partitioned numbers are written under one another:

47 + 76 =40 + 7
=70 + 6
110 + 13 = 123

375+567=300 + 70 + 5
500 + 60 + 7
800 + 130 + 12 = 942

Expanded column method

The expanded method leads children to the more compact column method so that they understand the structure and efficiency of it.

The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Example:
Write the numbers in columns:

Add the tens first		Add the ones first
47 + 76 110 13 123	Or	47 + 76 13 110 123

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to consistently adding the ones digits first.
The addition of the tens in the calculation 47 + 76 is described as 'Forty plus seventy equals one hundred and ten', stressing the link to the related fact 'Four plus seven equals eleven'.

Column method

The method is then shortened and when the column total is a two-digit number, the tens (or hundreds) are carried over into the next column. Use the words 'carry ten' or 'carry one hundred', **not** 'carry one'.

Example:

366
+ 458
824
1 1

Once learned, this method is quick and reliable.
Later, extend to adding three two-digit numbers, two three-digit numbers, and numbers with different numbers of digits. This method of can also be used to add decimals.

Progression in written methods for subtraction

These notes show the stages in building up to a compact, efficient written method for subtraction. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to:
- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (16 – 7) and their knowledge of place value
- know all complements to 10 and 100

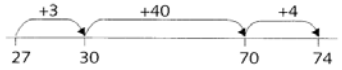
Children need to be able to:
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one
- partition numbers in different ways. e.g. 74 into 70 + 4 or 60 + 14
- subtract mentally a single-digit number or a multiple of 10 from a two-digit number
- add the totals (of the hundreds, tens and ones columns) mentally

Empty number line

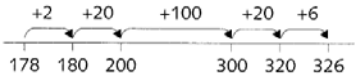
Empty or numbered lines are a useful way of modelling processes such as bridging through multiples of ten. The steps can be recorded by counting on or back.

Counting on example:

74 – 27 = 47



326 – 178 = 148



Counting back example:

15 – 7 = 8

74 – 27 = 47

The steps may be recorded in a different order or combined. With practice children will record less information and decide whether to count on or back.

Expanded column method

2 digit numbers with1 adjustment needed: 74 – 27 =

Partition into tens and ones. Then say, "There is not enough to subtract 7 from 4", rather than, "You ***can't*** subtract 7 from 4".

70 + 4 is then readjusted to become 60 + 14. The calculation can now be carried out.

$$\begin{array}{r} 70 + 4 \\ -\ 20 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 60 + 14 \\ -\ 20 + \ 7 \\ \hline 40 \ + \ 7 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {\scriptstyle 60 \quad 14} \\ \cancel{70} + \cancel{4} \\ -\ 20 + 7 \\ \hline 40 + 7 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {}^{6}\cancel{7}^{1}4 \\ -2\ 7 \\ \hline 4\ 7 \end{array}$$

Say, "60 – 20" or, "6 tens – 2 tens" not, "6 – 4"

3 digit numbers with1 adjustment needed: 563 – 271 =

Partition into hundreds, tens and ones.

500 + 60 needs to be adjusted to become 400 + 160. The calculation can now be carried out.

$$\begin{array}{r} 500 + 60 + 3 \\ -\ 200 + 70 + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 400 + 160 \ + \ 3 \\ -\ 200 + \ \ 70 \ + \ 1 \\ \hline 200 + \ \ 90 \ + \ 2 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {\scriptstyle 400 \quad 160} \\ \cancel{500} + \ \cancel{60} + 3 \\ -\ 200 + \ \ 70 + 1 \\ \hline 200 + \ \ 90 + 2 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {}^{4}\cancel{5}^{1}6\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Say, "400 – 200" or, "4 hundreds – 2 hundreds" not, "4 – 2"

3 digit numbers with 2 adjustments needed: 563 – 278 =

This occurs when the tens ***and*** the ones to be subtracted are larger than those you are subtracting from

Firstly, 60+3 is adjusted to become 50+13

500+50 is then adjusted to become 400+150. The calculation can now be carried out.

$$\begin{array}{r} 500 + 60 + 3 \\ -\ 200 + 70 + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 500 + 50 + 13 \\ -\ 200 + 70 + \ \ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 400 + 150 + 13 \\ -\ 200 + \ \ 70 + \ \ 8 \\ \hline 200 + \ \ 80 + \ \ 5 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {\scriptstyle \quad\quad 150} \\ {\scriptstyle 400 \quad \ 50 \quad 13} \\ \cancel{500} + \cancel{60} + \cancel{3} \\ -\ 200 + 70 + 8 \\ \hline 200 + 80 + 5 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {}^{4}\cancel{5}^{15}\cancel{6}^{\ 1}3 \\ -\ 2\ 7\ 8 \\ \hline 2\ 8\ 5 \end{array}$$

Say, "150 – 70" or "15 tens – 7 tens" not, "15 – 7"

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

When 0's are involved, the adjustments need to be done in a different order. There is not enough to subtract 8 from 3

As there are no tens, 500 + 0 is adjusted first to become 400 + 100

Then 100 + 3 can be adjusted to 90 + 13. The calculation can now be carried out.

$$\begin{array}{r} 500 + \ \ 0 \ + 3 \\ -\ 200 + 70 + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 400 + \ \ 100 + 3 \\ -\ 200 + \ \ 70 \ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 400 + \ \ 90 + 13 \\ -\ 200 + \ \ 70 + \ \ 8 \\ \hline 200 + \ \ 20 + \ \ 5 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {\scriptstyle \quad\quad 90 \quad 13} \\ {\scriptstyle 400 \quad 100} \\ \cancel{500} + \ \cancel{0} + \cancel{3} \\ -\ 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

Over time, recording is refined

$$\begin{array}{r} {}^{4}\cancel{5}^{91}\cancel{0}^{1}3 \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Say, "100 – 70" or, "10 tens – 7 tens" not, "10 – 7"

Progression in written methods for multiplication

These notes show the stages in building up to a compact, efficient method for multiplication. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

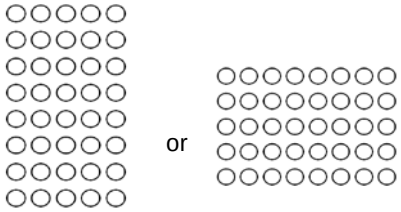
Children need to be able to:
- count in steps
- understand multiplication as repeated addition

Children need to be able to:
- partition numbers into multiples of one hundred, ten and one and in other ways
- recall multiplication facts to 10 × 10
- work out products such as 70 × 5, 70 × 50, 700 × 5, or 700 × 50, using the related fact, 7 × 5, and an understanding of place value
- add combinations of numbers mentally or using a written method

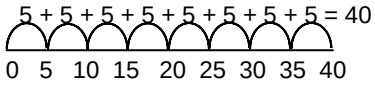
Method 1 – Repeated addition

Children start by understanding multiplication as arrays and repeated addition. They use this understanding to help them work out multiplication facts they cannot recall quickly

Example:
For '8 x 5', children picture:

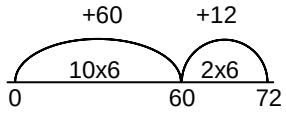


They use repeated addition to work out the calculation:



Recording of the steps on the number line may be refined as understanding and knowledge of facts develops:

Example:
12 x 6



This will support children in learning their tables using known facts and in understanding the distributive law which they will apply later when using the grid method.

Method 2 – Grid method into short/long multiplication

Multiplications can be carried out using the law of distribution which allows the numbers to be partitioned and each part to be multiplied separately. The products are then added to find the total product.

Stage 1 – Grid method

When multiplying a 1-digit number by a 2-digit number, children may choose to partition the numbers in different ways: 7 x 38

x	7
10	70
10	70
10	70
5	35
3	21
	266

x	7
10	70
10	70
10	70
8	56
	266

X	7
30	210
8	56
	266

Ensure that children understand the relationship between 7x3 and 7x30 and are not simply 'adding a nought'

The same method can also be applied when multiplying a 1-digit number by a 3-digit number:

x	6
500	3000
40	240
9	54
	3294

Ensure that children understand the relationship between 6x5 and 6x500 and are not simply 'adding 2 noughts'

When multiplying a 2-digit number by a 2-digit number:

(1) Partition both numbers and multiply each part

(2) Add the answers in each row

(3)Add the two row totals to find the final product

X	20	7
50	1000	350
6	120	42

X	20	7	
50	1000	350	1350
6	120	42	162

X	20	7	
50	1000	350	1350
6	120	42	162
			1512

Stage 2 – Short/long multiplication (only for most able)

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  38
 × 7
 56
210
266
```

Children describe what they are doing by referring to the value of the digits. Say, "30x7", not "3x7" although the relationship should be stressed

```
  549
 x 6
 54
240
3000
3294
```

Children say, "6x9, 6x40, 6x500"

```
  56
 × 27
 42
350
120
1000
1512
```

Children say, "7x6, 7x50, 20x6, 20x50"

Progression in written methods for division

These notes show the stages in building up to a compact, efficient method for division. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to:
- understand division as grouping and sharing
- understand multiplication and division as inverse operations
- recall multiplication and division facts to 10 × 10
- understand remainders
- derive larger multiples using known facts e.g. 10 x 3 = 30 ➔20 x 3 = 60 etc
- add multiples mentally and work out differences

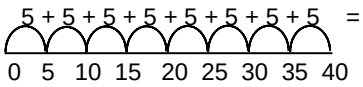
Stage 1 - Repeated addition

When it is not appropriate to use a sharing method for division and the division fact is not known, repeated addition (using the relationship between multiplication and division) can be used.

Example without remainder:

40 ÷ 5

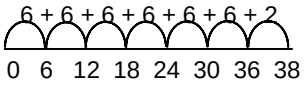
Ask "How many 5s in 40?"



= 8 fives

Example with remainder:

38 ÷ 6

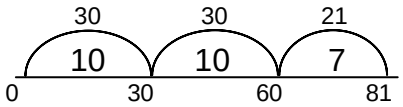


= 6 sixes with a remainder of 2

For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Example without remainder:

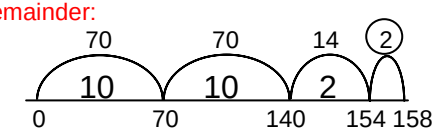
81 ÷ 3



This could either be done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether) or by counting in jumps of known multiples of 3 to reach 81 (30 + 30 + 21) then working out the number of threes in each jump.

Example with remainder:

158 ÷ 7



10 sevens are 70, add another 10 sevens is 140, add 2 more sevens is 154 add 2 makes 158. So there are 22 sevens with a remainder of 2.
The remainder is indicated above the jump rather than inside it, so that children do not mistakenly add 10, 10, 2 and 2 and get an answer of 24.

Children need to be able to:
- partition two-digit and three-digit numbers into multiples of one hundred, ten and ones in different ways
- recognise multiples of single-digit numbers
- derive larger multiples using known facts

Stage 2 - Expanded short division

A more compact recording of repeated addition is to use an expanded form of 'short' division. Here, the dividend is partitioned into multiples of the divisor (plus any remaining ones). Each part is then divided separately.
Expanded short division can be introduced to children who are confident with multiplication and division facts and whose understanding of partitioning and place value is sound.

Example without remainder:

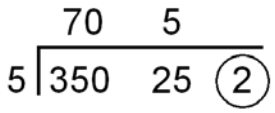
81 ÷ 3

81 is partitioned into multiples of 3 which are then each divided by 3	Over time, this is refined so that the dividend is partitioned into the highest multiple of the divisor that is also a multiple of 10, plus any remaining ones
10 + 10 + 7 3 ⟌ 30 + 30 + 21	20 + 7 3 ⟌ 60 + 21

Example with remainder:

377 ÷ 5

Children use the knowledge of 5 times tables to calculate that 350 = 70 x 5 and 400 = 80 x 5. Therefore 350 is the largest number (less that 377) which is a multiple of both 5 and 10. This leaves 27 which is then partitioned into 25 and 2.



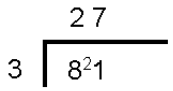
377 ÷ 5 = 75 remainder 2

Use with the most able children who have a secure understanding of all the previous steps.

Stage 3 - Short division

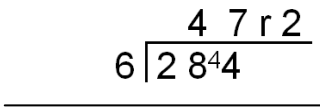
Example without remainder:

81 ÷ 3



Children use their knowledge of the 3 times table to find, "How many 3s in 80 where the answer is a multiple of 10?" This gives 20 threes (since 30 threes would be too many), with 20 remaining (2 tens are carried over to the next column) Now ask: 'How many threes in 21".

Example with remainder:



Once children's understanding of this method is secure they might shorten their dialogue to:

"How many 6s in 28?"
"4 remainder 4"
"How many 6s in 44?"
"7 remainder 2"

BUT ensure children have a secure understanding of what they are doing and are able to use their knowledge of related facts to either make a rough estimate first or have an idea about whether their final answer is reasonable or not.